

Rank of a matrix : No. of non zero rows in REF or RREF
 Notation $P(A)$

System of eqⁿs.

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2$$

$\vdots$

$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

$$AX = b$$

$m \times n, n \times 1$

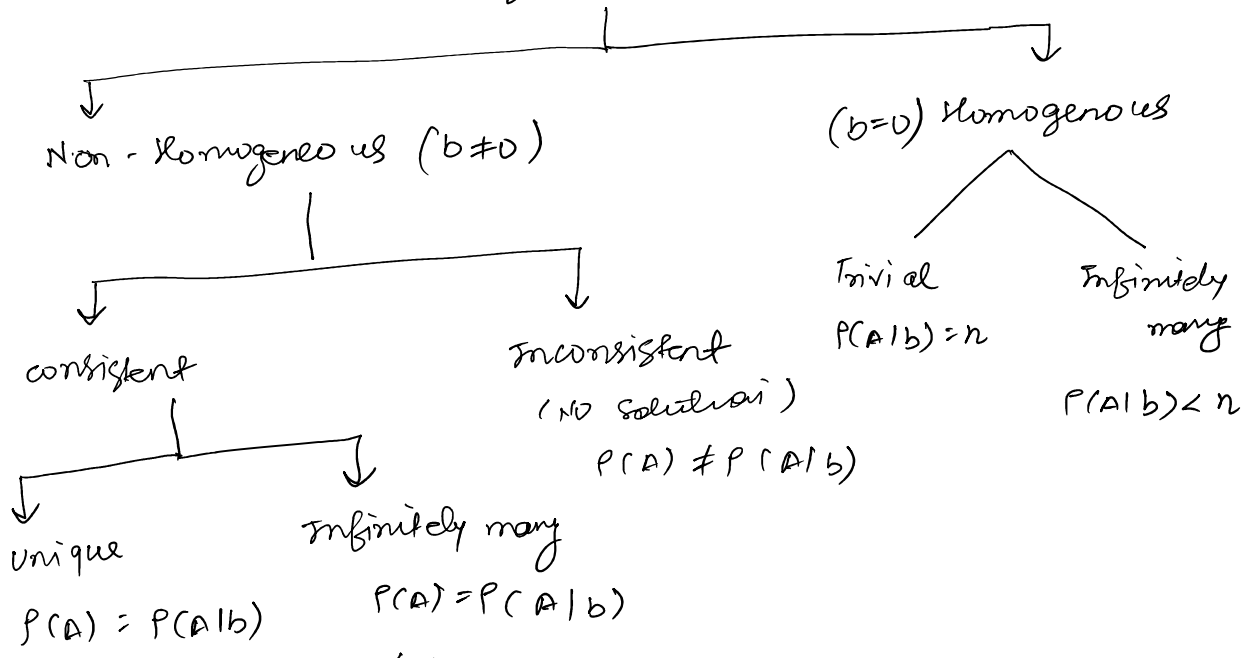
$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n \times 1}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}_{m \times 1}$$

$$[A|b] = \left[\begin{array}{cccc|c} a_{11} & \dots & a_{1n} & b_1 \\ a_{21} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & \dots & a_{mn} & b_m \end{array} \right] \rightarrow \text{Augmented matrix}$$

System of eqⁿ.



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$$P(A) = P(A|b)$$

$$P(A) = P(\overset{v}{A}|b)$$

$\leq n = \text{no. of variables}$

$$< n$$

Eg 1
$$\begin{bmatrix} 0 & 2 & 3 & 8 \\ 2 & 3 & 1 & 5 \\ 1 & -1 & -2 & -5 \end{bmatrix} \quad \text{REF} = \begin{bmatrix} 1 & -1 & -2 & -5 \\ 0 & 5 & 5 & 15 \\ 0 & 0 & 5 & 10 \end{bmatrix}$$

RREF
$$\begin{bmatrix} \textcircled{1} & -1 & -2 & -5 \\ 0 & \textcircled{1} & 1 & 3 \\ 0 & 0 & \textcircled{1} & 2 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\text{Rank} = 3$$

Eg
$$2y + 3z = 8, \quad 2x + 3y + z = 5, \quad x - y - 2z = -5$$

$$[A|b] = \begin{bmatrix} 0 & 2 & 3 & 8 \\ 2 & 3 & 1 & 5 \\ 1 & -1 & -2 & -5 \end{bmatrix}$$

$$P(A) = P(A|b) = 3 = \text{no. of variables}$$

1) Gauss Elimination method (REF)

2) Gauss Jordan method (RREF)

REF
$$\begin{bmatrix} x & y & z \\ 1 & -1 & -2 & -5 \\ 0 & 5 & 5 & 15 \\ 0 & 0 & 5 & 10 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{matrix} 5z = 10 \\ z = 2 \end{matrix}$$

$$5y + 5z = 15 \Rightarrow 5y = 5 \Rightarrow y = 1$$

$$x - y - 2z = -5 \Rightarrow x = 0$$

RREF
$$\begin{bmatrix} x & y & z \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad \begin{matrix} z = 2 \\ y = 1 \\ x = 0 \end{matrix}$$

Eg. 2) $w - x - y + 2z = 1, \quad 2w - 2x - y + 3z = 3, \quad -w + x - y = -3$

$$R \rightarrow R - 2R_1$$

Ex. 2) $w - x - y + 2z = 1$, $2w - x - y + z = 3$, $w - x - y + 2z = 1$

$$[A|b] = \begin{bmatrix} \textcircled{1} & -1 & -1 & 2 & 1 \\ 2 & -2 & -1 & 3 & 3 \\ -1 & 1 & -1 & 0 & -3 \end{bmatrix} \quad \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 + R_1 \end{array}$$

$$= \begin{bmatrix} \textcircled{1} & -1 & -1 & 2 & 1 \\ 0 & 0 & \textcircled{+1} & -1 & 1 \\ 0 & 0 & -2 & 2 & -2 \end{bmatrix} \quad \begin{array}{l} \text{REF.} \\ R_1 \rightarrow R_1 + R_2 \\ R_3 \rightarrow 2R_2 + R_3 \end{array}$$

$w \quad x \quad y \quad z$

$$= \begin{bmatrix} \textcircled{1} & -1 & 0 & 1 & 2 \\ 0 & 0 & \textcircled{1} & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \rightarrow \text{RREF}$$

$$\rho = \rho(A) = \rho(A|b) < n (=4)$$

x & z are free variables. $\underline{x = k_1 \quad z = k_2}$

$$y - z = 1 \Rightarrow y = \underline{1 + k_2}$$

$$w - x + z = 2 \Rightarrow \underline{w = 2 - k_2 + k_1}$$